Given that

$$\begin{aligned}
\mathrm{p}(\boldsymbol{w}|\mathsf{t}) &= \mathcal{N}(\boldsymbol{w}|\boldsymbol{m}_N, \boldsymbol{S}_N) \\
\mathrm{p}(t_{N+1}|\boldsymbol{w}) &= \mathcal{N}(t_{N+1}|\boldsymbol{w}^\top \boldsymbol{\phi}(\boldsymbol{x}_{N+1}), \beta^{-1}) \\
&= \mathcal{N}(t_{N+1}|\boldsymbol{\phi}(\boldsymbol{x}_{N+1})^\top \boldsymbol{w}, \beta^{-1})
\end{aligned} \tag{3.49}$$

and the general result for linear-Gaussian models given by Equation (2.116), we have

$$\begin{aligned}
\boldsymbol{S}_{N+1} &= [\boldsymbol{S}_N^{-1} + \beta \boldsymbol{\phi}(\boldsymbol{x}_{N+1}) \boldsymbol{\phi}(\boldsymbol{x}_{N+1})^\top]^{-1} \\
\boldsymbol{S}_{N+1} &= \boldsymbol{S}_N + [\beta \boldsymbol{\phi}(\boldsymbol{x}_{N+1}) \boldsymbol{\phi}(\boldsymbol{x}_{N+1})^\top]^{-1} \\
\mathrm{p}(\boldsymbol{w}|\mathsf{t}, t_{N+1}) &= \mathcal{N}(\boldsymbol{w}|\boldsymbol{S}_{N+1}[t_{N+1} \beta \boldsymbol{\phi}(\boldsymbol{x}_{N+1}) + \boldsymbol{S}_N^{-1} \boldsymbol{m}_N], \boldsymbol{S}_{N+1})
\end{aligned}$$

■