

Given that

$$E_D = \frac{1}{2} \sum_{n=1}^N r_n \{t_n - \mathbf{w}^\top \phi(\mathbf{x}_n)\}^2 \quad (3.104)$$

we find the \mathbf{w}_{ML} by searching the point nulling the partial derivative of E_D w.r.t \mathbf{w}

$$\frac{\partial E_D}{\partial \mathbf{w}} = \sum_{n=1}^N r_n \mathbf{w}^\top \phi(\mathbf{x}_n) \phi(\mathbf{x}_n)^\top - r_n t_n \phi(\mathbf{x}_n)^\top$$

$$\sum_{n=1}^N r_n \mathbf{w}_{\text{ML}}^\top \phi(\mathbf{x}_n) \phi(\mathbf{x}_n)^\top - r_n t_n \phi(\mathbf{x}_n)^\top = \mathbf{0}$$

$$\Leftrightarrow \mathbf{w}_{\text{ML}}^\top \sum_{n=1}^N \phi(\mathbf{x}_n) r_n \phi(\mathbf{x}_n)^\top = \sum_{n=1}^N r_n t_n \phi(\mathbf{x}_n)^\top$$

$$\Leftrightarrow \mathbf{w}_{\text{ML}}^\top \Phi^\top \text{diag}(\mathbf{r}) \Phi = \mathbf{t}^\top \text{diag}(\mathbf{r}) \Phi$$

$$\Leftrightarrow \Phi^\top \text{diag}(\mathbf{r}) \Phi \mathbf{w}_{\text{ML}} = \Phi^\top \text{diag}(\mathbf{r}) \mathbf{t}$$

$$\Leftrightarrow \mathbf{w}_{\text{ML}} = (\Phi^\top \text{diag}(\mathbf{r}) \Phi)^{-1} \Phi^\top \text{diag}(\mathbf{r}) \mathbf{t}$$

From equations (3.11-12), the noise variance of \mathbf{x}_n is scaled by r_n from β^{-1} to $(\beta r_n)^{-1}$. Also, one can interpret it as a leverage of data point's importance by attributing a larger weight to frequently appearing data. ■