

$$E(\mathbf{w}) = \frac{1}{2} \sum_{n=1}^N \left(\sum_{j=0}^M w_j x_n^j - t_n \right)^2$$

This is a sum of quadratic functions of \mathbf{w} , as such E is convex in \mathbf{w} . To minimize the error, we derive $E(\mathbf{w})$ with respect to \mathbf{w} , and find the root:

$$\begin{aligned} \frac{\partial E(\mathbf{w})}{\partial w_i} = 0 &\iff \sum_{n=1}^N \left(\sum_{j=0}^M w_j x_n^j - t_n \right) x_n^i = 0 \\ &\iff \sum_{n=1}^N \sum_{j=0}^M w_j x_n^j x_n^i = \sum_{n=1}^N t_n x_n^i \\ &\iff \sum_{j=0}^M \left(\sum_{n=1}^N x_n^{j+i} \right) w_j = \sum_{n=1}^N t_n x_n^i \\ &\iff \sum_{j=0}^M A_{ij} w_j = T_i \end{aligned}$$

Where:

$$A_{ij} = \sum_{n=1}^N x_n^{j+i} \quad T_i = \sum_{n=1}^N t_n x_n^i$$

